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errors of R.A. are negative in the earlier years, and positive in the later years, as if the mean motion required still further correction. It must be pointed out, however, that the comparison between the observed and computed places has not been made with any attempt at great accuracy, but rather as a general check on the correctness of the work; and that, in particular, the perturbations have not been recomputed as, of course, they ought to have been had the intention been to deduce a *definitive* orbit of the planet. To that the present paper makes no pretension, the object being merely to deduce corrections to the assumed elements of the orbit so as to obtain elements sufficiently accurate for the purpose of computing ephemerides for some years to come. And, as far as can be seen, that object has been attained.

My thanks are due to the Council of the Royal Society for a grant to defray the expenses of the computations, the results of which are embodied in this paper.

Blackheath:
1890 *June* 4.

On Star-Correction Tables. By W. H. Finlay, M.A.

The computation of star-corrections forms no small part of the reductions for the formation of a catalogue, as every astronomer who has taken part in such work knows well; so that any attempt to shorten the time required for these computations, without sacrificing accuracy or simplicity, is worthy of consideration. I think that this may be satisfactorily done to a considerable extent by tables in the following way. We have for any star—

$$\alpha' - \alpha = f + g \sin(G+\alpha) \tan\delta + h \sin(H+\alpha) \sec\delta,$$

$$\delta' - \delta = \iota \cos\delta + g \cos(G+\alpha) + h \cos(H+\alpha) \sin\delta.$$

Now I put—

$$P = \tfrac{1}{15}\cdot g_0 \sin(G+\alpha) \tan\delta, \qquad P' = g_0 \cos(G+\alpha),$$

$$Q = \tfrac{1}{15}\cdot h_0 \sin(H+\alpha) \sec\delta, \qquad Q' = h_0 \cos(H+\alpha) \sin\delta,$$

and

$$I = \iota \cos\delta,$$

where g_0 and h_0 are arbitrarily chosen constants; and I tabulate the values of P, Q, Q′ for each degree of declination and for every two minutes of time. The value of P′, which is independent of the declination, is given in a separate table for every minute (or smaller interval if desired) of time.

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The star-corrections for any given date are then—

$$\alpha' - \alpha = \tfrac{1}{15}.f + \text{P}.\frac{g}{g_0} + \text{Q}.\frac{h}{h_0} = \tfrac{1}{15}.f + \text{P}\left(1 + \frac{g-g_0}{g_0}\right) + \text{Q}\left(1 + \frac{h-h_0}{h_0}\right)$$

(in time)

$$= \tfrac{1}{15}.f + \text{P}(1+x) + \text{Q}(1+y),$$

and, similarly,

$$\delta' - \delta = \text{I} + \text{P}'(1+x) + \text{Q}'(1+y),$$

where

$$x = \frac{g-g_0}{g_0}, \text{ and } y = \frac{h-h_0}{h_0}.$$

I is also tabulated for each degree of declination for values of ι, proceeding by differences of 0″·1.

The values chosen for g_0 and h_0 are 20″·053 and 18″·5, respectively, so that x is generally negative and never greater numerically than unity, while y is always positive and never greater than 0·11. The terms Px, P′x, Qy and Q′y can therefore be easily taken from Crelle's tables.

P, Q, P′, Q′ are taken from the tables with the argument G + α for P and P′, and H + α for Q and Q′. The values for the nearest whole minute of time will generally be amply sufficient in accuracy, but closer interpolation (if desired) is very simple. The increments in the values of P, Q, and Q′ for 1° of declination are given down the columns, and their variations for every fraction of a degree can be written at once under their tabular values. The signs of the terms are taken from a small table with the arguments G + α and H + α; the upper sign, where two occur together, being for northern, and the lower for southern, declinations.

As an example, I give a portion of the table for declination 40°, in which the small figures at the side of each column give the change of the function for 1° of declination:—

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40°

0.—VI.	±	+	+	±
VI.—XII.	±	−	+	∓

	G+α	H+α			
	P	Q	Q′		
	s	s	″		
	…	…	…	…	
I. XIII. ↓	0	0·290	0·417	11·49	0 {XI. XXIII. ↑
	2	0·300	0·430	11·46^{24}	58
	4	0·309^{11}	0·444^{7}	11·43	56
	6	0·319	0·457	11·40	54
	8	0·328	0·471	11·37	52
	10	0·337^{12}	0·484	11·34	50
	12	0·347	0·498	11·31^{23}	48
	14	0·356^{13}	0·511	11·28	46
	16	0·365	0·524^{8}	11·24	44
	18	0·374	0·537	11·21	42
	20	0·384	0·551	11·17	40
	22	0·393^{14}	0·564	11·14	38
	24	0·402	0·577	11·10	36
	26	0·411	0·590	11·06	34
	28	0·420	0·603^{9}	11·03	32
	…	…^{15}	…	…	…
	…	…	…	…	… X. XXII. ↑

		G+α	H+α		
		P	Q	Q′	
		s	s	″	
	…	…	…	…	…
IV. XVI. } ↓	0	0·971	1·394	5·95	0 {VIII. XX. ↑
	2	0·976^{35}	1·401^{21}	5·86^{12}	58
	4	0·981	1·408	5·77	56
	6	0·986	1·415	5·67	54
	8	0·990	1·422	5·58	52
	10	0·995^{36}	1·428	5·49	50
	12	0·999	1·435	5·40^{11}	48
	…	…	…^{22}	…	…
	…	…	…	…	…
	40	1·054	1·513	4·07	20
	42	1·057^{38}	1·518^{23}	3·97^{8}	18
	44	1·061	1·522	3·87	16
	46	1·064	1·527	3·77	14
	48	1·067	1·531	3·67	12
	50	1·070	1·535	3·58	10
	…	…	…	…	… {VII. XIX. ↑

	P	P′	Q	Q′
XII.—XVIII.	∓	−	−	∓
XVIII.—XXIV.	∓	+	+	±

ι	I
″	″
…	…
3·6	2·76
3·7	2·83^{-4}
3·8	2·91
3·9	2·99
4·0	3·06
4·1	3·14
4·2	3·22
4·3	3·29^{-5}
…	…
…	…
7·8	5·97
7·9	6·05^{-9}
8·0	6·13
8·1	6·20
8·2	6·28

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Now with this table let us find the star-corrections for—

Ex. 1. β Bootis, 1889 May 20.

From the American Ephemeris we get for this date—

$$f\ (\textit{in time}) = +0^{s}{\cdot}114,\ G = 4^{h}\ 52^{m}{\cdot}3,\ H = 13^{h}\ 50^{m}{\cdot}5,\ \iota = -4''{\cdot}03,$$

and—

x is found to be $= -{\cdot}869$, $y = +{\cdot}084$.

The computation is then as follows:—

1889 *May* 20.

	h m		h m		° ′
$\alpha =$	14 57·8	$\alpha =$	14 57·8	$\delta =$	+40 50
$G =$	4 52·3	$H =$	13 50·5		
$G+\alpha =$	19 50·1	$H+\alpha =$	4 48·3		

R.A.	s		Dec.	″	
P_0	0·995		P_0'	9·26 (*from separate table*)	
δP	30		$P'x$	−8·05	
sum	1·025				
Px	−·891				
		(1) = −0·134			(1) = +1·21
Q_0	1·532		Q_0'	3·67	
δQ	19		$\delta Q'$	6	
Qy	+130		$Q'y$	+31	
		(2) = +1·681			(2) = +4·04
f		(3) = +0·114	I	3·09 − ·03	(3) = −3·06
sum		+1·661	sum		+2·19

Ex. 2. * 1889 March 12.

For this date—

$$f = -0^{s}{\cdot}369,\ G = 9^{h}\ 5^{m}{\cdot}0,\ H = 18^{h}\ 31^{m}{\cdot}1,\ \iota = -8''{\cdot}08,$$

and—

x is found to be $= -{\cdot}835$, $y = +{\cdot}015$.

	h m		h m		° ′
$\alpha =$	4 8·1	$\alpha =$	4 8·1	$\delta =$	−40 24
$G =$	9 5·0	$H =$	18 31·1		
$G+\alpha =$	13 13·1	$H+\alpha =$	22 39·2		

R.A.	s		Dec.	″	
P_0	0·352		P_0'	19·04 (*from separate table*)	
δP	5		$P'x$	−15·90	
sum	·357				
Px	−·298				
		(1) = +0·059			(1) = −3·14
Q_0	0·557		Q_0'	11·16	
δQ	3		$\delta Q'$	9	
Qy	8		$Q'y$	17	
		(2) = −0·568			(2) = −11·42
f		(3) = −0·369	I	6·19 − ·04	(3) = −6·15
sum		−0·878	sum		−20·71

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The signs are not to be placed before the quantities as taken from the tables, but are inserted before the final values in the last column—viz. after (1), (2), and (3); the sign of I is always the same as that of ι. The quantities δP, δQ, δQ′ are the increments of P, Q, Q′ for the minutes of declination (in our case 50 for Ex. 1, 24 for Ex. 2), and are always additive if P, Q, and Q′ be taken from the table for the nearest degree *numerically* less than the star's declination, while the correction to I is subtractive.

A computation of one pair of these star-corrections with four-figure logarithms would require 160 figures and symbols; the ordinary method (Aa+Bb+&c.) requires 217 figures, without taking into account the labour of forming a, b, &c.; while the result above has been obtained with 110 figures. This means a very considerable saving of time when we consider that some thousands of these computations have to be made every year. It might be more convenient, in the case of only a few star-corrections being wanted, to have the values of P′ given in the tables with the others; but if many stars are being reduced, the first step would be to run in the values of P′ for all of them as soon as the values of G+a are formed.

At declination 60° the change of P for 1° is $0^{s}{\cdot}096$, which is perhaps too large for δP to be readily written down; the change of Q is somewhat less; so that it may be advisable to reduce the intervals of declination for which the tables, after 50°, are computed to half degrees. As a matter of fact, the star-corrections are practically only required to the nearest hundredth in R.A. and the nearest tenth in declination; and I do not anticipate any difficulty in using the tables as far as 70°. They will certainly save much time when the declination is small.

The value chosen for g_0 is the constant n of the precession for the year 1900, so that for many years the precessions in right ascension and declination for any star can be taken at once from the proper declination table with the argument R.A. —viz. $3^{s}{\cdot}073$+P and P′, due regard being paid to the signs of P and P′. But, as far as concerns the computation of star-corrections, the tables hold good for all time, any changes in the astronomical constants being taken account of in the data of the Ephemeris and in the values of x and y.

For the most effective use of the tables the values of f, G, H should be given in time and ι in seconds of arc (as is done in the American Ephemeris), and the values of x and y for each day in the year. These two last might, however, be readily taken from a table proceeding by increments of $0''{\cdot}1$ of g and h.

I may mention that approximate tables of this kind were prepared by means of Crelle's tables, and used for a check-computation of the star-corrections for the *Victoria* and *Sappho* stars observed here last year, and were found to be satisfactory; but in order to secure accuracy in the last decimal place of the

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tables a more exact calculation is necessary. I have made considerable progress in the calculation of accurate tables, and hope soon to have them completed.

Cape of Good Hope :
1890 *May* 5.

On the Computation of the Equation of the Centre in Elliptical Orbits of Moderate Eccentricities. By A. Marth.

Though the indirect solution of Kepler's problem is a sufficiently easy proceeding when only a limited number of positions have to be computed, it takes rather too much time and becomes tiresome when it has to be repeated periodically hundreds of times. To provide for such cases some method is required for deducing the true anomalies directly from the mean anomalies with the desired degree of accuracy.

To find the true anomalies with the accuracy sufficient for approximate ephemerides of the minor planets, two sets of general tables have been computed: Mr. Godward's "Auxiliary Tables for Computing an Approximate Ephemeris of a Minor Planet" . . . printed 1866, and used in the *Nautical Almanac* office, and MM. Callandreau and Fabry's "Tables numériques destinées a faciliter le calcul des éphémerides" . . . published, 1885, in the second volume of the *Bulletin Astronomique.*

To get the true anomalies with an accuracy required for closer computations, the late E. Schubert gives in his later tables and papers on the elements of the minor planets a special "Table for the correction to be added to the auxiliary anomaly v'," this v' being found by the equation

$$\cot \tfrac{1}{2} v' = \frac{1-e}{1+e} \cot \tfrac{1}{2} \text{ mean anomaly.}$$

The present paper is intended to furnish the means for facilitating the calculation of certain tables, which will allow the differences between the true and mean anomalies to be easily deduced, whenever the intended work may make the construction of such tables desirable.

If e denotes the eccentricity, μ and v the mean and true anomaly, ω the angular value of the radius, the equation of the centre is given by the known series (all terms to e^{10} inclusive being retained) as follows :—